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PERFORMANCE ANALYSIS OF NONCOHERENT DIFFERENTIAL PHASE SHIFT KEYING USING POST-DETECTION SELECTION COMBINING OVER A RAYLEIGH FADING CHANNEL

by

Tahir Conka

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Thesis Advisor:	Tri T. Ha
Co-Advisor:	Ralph D. Hippenstiel

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Post-Detection Selection Combining (PDSC) is evaluated and compared to Equal Gain Combining (EGC) and Selection Combining (SC), the two common diversity techniques discussed in the literature.

Numerical results obtained for Post-Detection Selection Combining are compared to Selection Combining and Equal Gain Combining. The Post-Detection Selection Combining method is shown to be superior to the Selection Combining method but inferior to Equal Gain Combining method for a non-coherent DPSK receiver operating over a Rayleigh fading channel.

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Tahir Conka
Lieutenant Junior Grade, Turkish Navy
B.S., Naval Postgraduate School, 1997

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Author: Tahir Conka
Tahir Conka

Approved by: Tri T. Ha
Tri T. Ha, Thesis Advisor

Ralph D. Hippenstiel
Ralph D. Hippenstiel, Co Advisor

Herschel H. Loomis, Jr.
Herschel H. Loomis, Jr., Chairman, Department of Electrical
and Computer Engineering

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I. INTRODUCTION

A. BACKGROUND

The use of a natural medium for communications systems, especially for wireless and mobile radio communications systems, implies unavoidable degradation associated with the randomness that accompanies natural phenomena. Refraction, reflection, diffraction, scattering, focusing attenuation and other factors cause variations between the received signal level and a calculated free-space level for a given transmitter output. The degradation is usually due to natural causes but can also be caused by man made items such as huge buildings in an urban area [1,2].

Although only the direct wave is desired, several distinguishable paths between the transmitter and receiver do exist. These are typically due to reflections. These paths cause several waves to arrive at the receiver at slightly different times and produce fading. The interference between a direct wave and a reflected wave is called multi-path fading [2].

The electro-magnetic waves tend to arrive at the receiver over different paths having different propagation delays. The received signal consists of components having randomly distributed amplitudes, phases and angles of arrival. When there is no single line-of-sight path between the transmitter and the receiver, the received signal is modeled as a Rayleigh random variable. Its probability density function (pdf) is given by [4]

$$f(z)=\begin{cases}\dfrac{z}{\sigma^2}\exp\left(-\dfrac{z}{2\sigma^2}\right) & z\geq 0\\ 0 & otherwise\end{cases}, \qquad (1)$$

where $2\sigma^2$ is the average power of faded signal, and z is the amplitude of the received signal.

The resulting communication channel is called a Rayleigh fading channel. When the line-of-sight component is not zero, the received signal is modeled as a Ricean random variable. Its pdf is given by [4]

$$f(z)=\begin{cases}\dfrac{z}{\sigma^2}\exp\left(-\dfrac{z^2+a^2}{2\sigma^2}\right)I_0\left(\dfrac{a\,z}{\sigma^2}\right) & a\geq 0, z\geq 0 \\ 0 & otherwise\end{cases}\quad, \tag{2}$$

where a^2 is the average power of the unfaded (direct) signal component, $2\sigma^2$ is the average power of the faded signal component and $I_0(\bullet)$ is the zeroth-order modified Bessel function of the first kind. This resulting communication channel is called a Ricean fading channel [2,3].

Rayleigh fading severely degrades the average probability of the bit error performance of the receiver. One way of minimizing this problem is to increase the transmitter power or the antenna size. However, such a solution requires costly engineering alterations making other alternatives very attractive. It is well known, that diversity combining is one of the most effective alternatives [1].

To reduce the effects of fading, one of several diversity techniques can be implemented. Diversity is a procedure to receive or transmit the same symbol multiple times in order to provide redundancy at the receiver. The basic idea of diversity is that some of the received redundant symbols will be more reliable than the others, and the demodulation decision will be made using the more reliable symbols. In order to be useful, each redundant symbol must be received independently [4]. The diversity can be implemented as space, time, frequency, angle and polarization diversity. Space diversity consists of using multiple antennas at the receiver in order to receive the transmitted symbol multiple times via multiple paths. Time diversity is employed by transmitting the same symbol multiple times. In frequency diversity, the symbol is transmitted on multiple carrier frequencies at the same time. Angle diversity is implemented by a set of directive antennas where each one responds independently to a wave to produce an uncorrelated faded signal. Polarization diversity is possible with two orthogonal polarizations. Combining the signals from the diversity branches allows a significant performance improvement. Diversity combining can be accomplished prior to or after detection, which are called pre- and post-detection, respectively. Diversity combining can be linear or non-linear. In this thesis, only linear combining methods are analyzed. Unlike the other methods used for reducing fading effects, diversity combining lowers costs and reduces complexity in the receiver [2]. In this thesis, a new method referred to as Post-Detection Selection Combining (PDSC) is presented and compared with Selection Combining and Equal Gain Combining [1,3].

Post-Detection Selection Combining (PDSC) is invented by Professors Tri T. Ha and Ralph D. Hippenstiel [14]. This method selects the signal with the largest amplitude or combines the two (three) received signals with the two (three) largest amplitudes after detection. Since combining is done after detection, the new method is named as First, Second, and Third Order Post-Detection Selection Combining and is denoted by PDSC-1, PDSC-2 and PDSC-3, respectively.

B. SYSTEM DESCRIPTION

A noncoherent differential phase shift keyed (DPSK) receiver, as shown in Fig. 1, is used to demonstrate the performance of Post-Detection Selection Combining. As a noncoherent receiver, the DPSK receiver requires no phase recovery and can be build inexpensively. For this reason, it is widely used in wireless communications systems.

While DPSK signaling has the advantage of reduced receiver complexity, its energy efficiency is inferior to that of coherent PSK by about 3 dB [4]. The average probability of error for DPSK in additive white Gaussian noise is given by [4]

$$P_e = \frac{1}{2}\exp\left(\frac{-E_b}{N_0}\right) , \tag{3}$$

where E_b / N_0 is the bit-energy-to-noise-density ratio.

Before we analyze the Post-Detection Selection Combining, we need to consider general structure of the DPSK receiver. The phase difference between two successive bits is used to convey information. Whenever the data bit $b_i = 0$ is transmitted, the waveform for two consecutive differentially encoded bits $c_{i-1}c_i$ is given by [6]

$$\begin{aligned} v^{(1)}(t) &= \pm A\, p_T(t-(i-1)T)\cos(2\pi f_c t+\theta) \pm A\, p_T(t-iT)\cos(2\pi f_c t+\theta) \\ &= [\pm A\, p_T(t-(i-1)T) \pm A\, p_T(t-iT)]\cos(2\pi f_c t+\theta). \end{aligned} \tag{4}$$

When the data bit $b_i = 1$ is transmitted, the waveform, which has a phase change of π radians between two consecutive bits, is given by

$$\begin{aligned} v^{(2)}(t) &= \pm A\, p_T(t-(i-1)T)\cos(2\pi f_c t+\theta) \mp A\, p_T(t-iT)\cos(2\pi f_c t+\theta) \\ &= [\pm A\, p_T(t-(i-1)T) \mp A\, p_T(t-iT)]\cos(2\pi f_c t+\theta), \end{aligned} \tag{5}$$

where θ is the signal phase, f_c is the carrier frequency, A is the amplitude and the $\pm$ signs denotes the polarity of the two consecutive bits. The pulse $p_T(t-iT)$ is given by

$$p_T(t-iT) = \begin{Bmatrix} 1 \ , & iT < t \le (i+1)T \\ 0 \ , & otherwise \end{Bmatrix}. \tag{6}$$

When $b_1 = 0$ is transmitted, the received signal is of the form

$$r(t)=\left[\pm A\, p_T(t)\ \pm A\, p_T\,(t-T)\right]\cos\left(2\pi\, f_c\, t+\theta\right)+n(t)\,, \tag{7}$$

where n(t) is additive white Gaussian noise with zero mean and power spectral density $\frac{N_0}{2}$.

The in-phase outputs of the receiver are given by

$$Y_{1c}=\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{'}\right)+\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{''}\right)=\pm AT\cos\theta\ +N_{1c}\,, \tag{8}$$

$$Y_{2c}=\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{'}\right)-\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{''}\right)=N_{2c}\,,$$

where

$$N_{1c}=N_{1c}{'}+N_{1c}{''}\,,\quad N_{2c}=N_{1c}{'}-N_{1c}{''}\,, \tag{9}$$

and

$$N_{1c}{'}=\int_0^T n(t)\cos(2\pi\, f_c\, t)dt\,, \tag{10}$$

$$N_{1c}{''}=\int_T^{2T} n(t)\cos(2\pi\, f_c\, t)dt\,.$$

The quadrature outputs of the receiver are obtained as

$$Y_{1s}=\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{'}\right)+\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{''}\right)=\mp AT\sin\theta\ +N_{1s}\,, \tag{11}$$

$$Y_{2s}=\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{'}\right)-\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{''}\right)=N_{2s}\,,$$

where

$$N_{1s}=N_{1s}{'}+N_{1s}{''}\,,\quad N_{2s}=N_{1s}{''}-N_{1s}{''}\,, \tag{12}$$

and

$$N_{1s}{'}=\int_0^T n(t)\sin(2\pi\, f_c\, t)dt\,, \tag{13}$$

$$N_{1s}{''}=\int_T^{2T} n(t)\sin(2\pi\, f_c\, t)dt\,.$$

N_{1c}, N_{2c}, N_{1s} and N_{2s} are independent, identically distributed (i.i.d.) Gaussian random variables with zero mean and variance equal to $\dfrac{N_0T}{2}$.

Assuming that $b_1 = 0$ is transmitted, the outputs of the signal branch are

$$\begin{aligned} Y_{1c} &= \pm AT\cos\theta + N_{1c}, \\ Y_{1s} &= \mp AT\sin\theta + N_{1s}, \end{aligned} \tag{14}$$

and the outputs of the non-signal branch are

$$\begin{aligned} Y_{2c} &= N_{2c}, \\ Y_{2s} &= N_{2s}. \end{aligned} \tag{15}$$

As seen in Fig. 1, the decision variable for the signal branch is given by

$$Y_{1c}{}^2 + Y_{1s}{}^2 = (\pm AT\cos\theta + N_{1c})^2 + (\mp AT\sin\theta + N_{1s})^2. \tag{16}$$

For the non-signal branch, the corresponding variable is given by

$$Y_{2c}{}^2 + Y_{2s}{}^2 = N_{2c}{}^2 + N_{2s}{}^2. \tag{17}$$

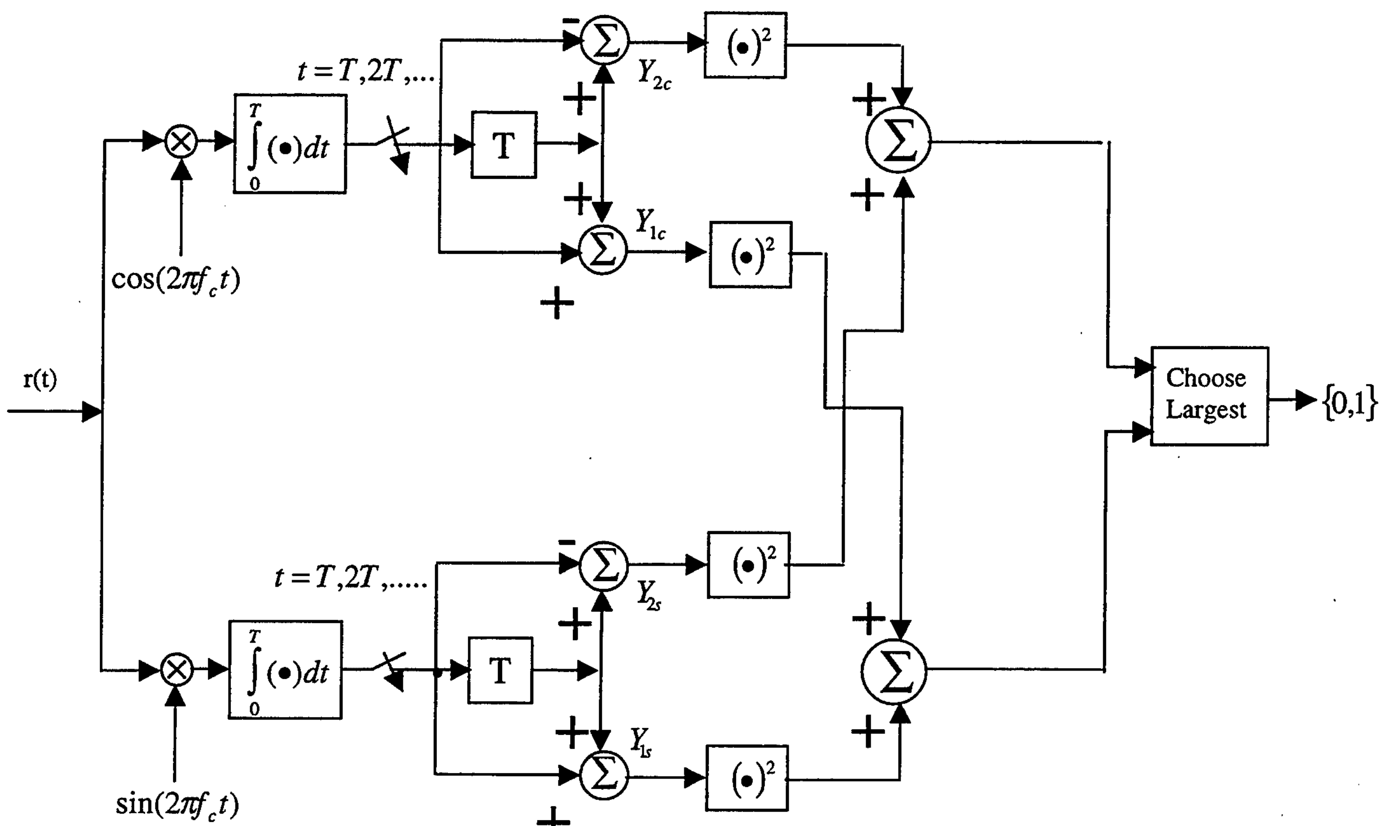

Figure 1. Block diagram of a noncoherent DPSK receiver

For convenience , we normalize (16) as follows:

$$
\begin{aligned}
Y_{1c}^2 + Y_{1s}^2 &= \left[\sqrt{\frac{T}{2}}\left(\pm\sqrt{2T}A\cos\theta + \frac{\sqrt{2}}{\sqrt{T}}N_{1C}\right)\right]^2 + \left[\sqrt{\frac{T}{2}}\left(\mp\sqrt{2T}A\sin\theta + \frac{\sqrt{2}}{\sqrt{T}}N_{1S}\right)\right]^2 \\
&= \frac{T}{2}\left(\pm\sqrt{2T}A\cos\theta + \frac{\sqrt{2}}{\sqrt{T}}N_{1C}\right)^2 + \frac{T}{2}\left(\mp\sqrt{2T}A\sin\theta + \frac{\sqrt{2}}{\sqrt{T}}N_{1S}\right)^2 \quad .
\end{aligned}
\tag{18}
$$

By setting $Y_1 = \frac{2}{T}\left(Y_{1c}^2 + Y_{1s}^2\right)$, $n_{1c} = \sqrt{\frac{2}{T}}N_{1C}$, $n_{1s} = \sqrt{\frac{2}{T}}N_{1s}$, $A' = \pm\frac{\sqrt{2T}}{2}A$, we have the decision variable

$$
Y_1 = \left(2A'\cos\theta + n_{1c}\right)^2 + \left(-2A'\sin\theta + n_{1s}\right)^2 . \tag{19}
$$

For a Rayleigh fading channel, A' is a Rayleigh random variable with parameter σ as given in (1). Furthermore, for a uniform variable θ, $A'\cos\theta$ and $-A'\sin\theta$ are Gaussian random variables with zero mean and variance σ^2 [11]. The variables $2A'\cos\theta$ and $-2A'\sin\theta$ are both zero mean Gaussian random variables with variance $4\sigma^2$. The zero mean Gaussian random variable n_{1c} and n_{1s} are independent with variance $\sigma_n^2 = N_0$. Therefore the random variable Y_1 (19) is a central chi-square random variable with the following density function [4]

$$
f_{Y_1}(y_1) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{y_1}{2\sigma_1^2}\right), \qquad y_1 \geq 0, \tag{20}
$$

where

$$
\sigma_1^2 = 4\sigma^2 + \sigma_n^2 = 4\sigma^2 + N_0 . \tag{21}
$$

Define the average energy per diversity channel as

$$
\overline{E} = 2\sigma^2 ; \tag{22}
$$

then, for L diversity channels, the bit energy is

$$\overline{E_b} = L\overline{E}. \tag{23}$$

For the non-signal branch, the decision variable $Y_2 = \frac{2}{T}\left(Y_{2c}^2 + Y_{2s}^2\right)$ can be written as

$$Y_2 = n_{2c}^2 + n_{2s}^2, \tag{24}$$

where $n_{2c} = \sqrt{\frac{2}{T}}N_{2c}$ and $n_{2s} = \sqrt{\frac{2}{T}}N_{2s}$ are zero mean Gaussian random variables with variance $\sigma_n^2 = N_0$. The density function of Y_2 is a central chi-square density function [4]

$$f_{Y_2}(y_2) = \frac{1}{2\sigma_2^2}\exp\left(-\frac{y_2}{2\sigma_2^2}\right), \qquad y_2 \geq 0, \tag{25}$$

where

$$\sigma_2^2 = \sigma_n^2 = N_0. \tag{26}$$

C. OBJECTIVE

In the following chapters, probability density functions are obtained and the bit error probabilities are derived for PDSC-1, PDSC-2, and PDSC-3. In Chapter VI, Post-Detection Selection Combining, Selection Combining and Equal Gain Combining are compared.

The primary objectives are to

(a) present the new Post-Detection Selection Combining method,

(b) make the necessary bit error rate derivations, and

(c) compare the performances of Post-Detection Selection Combining, Selection Combining and Equal Gain Combining.

II. REVIEW OF PREVIOUS WORK

A. EQUAL GAIN COMBINING

The Equal Gain Combining (EGC) method adds in a noncoherent fashion equally weighted branch signals. Although EGC is one of the most commonly used diversity techniques, the receiver is dependent on the order of the diversity [3]. The bit error probability expression for Equal Gain Combining is given by [4]

$$P_b = \sum_{k=0}^{L-1} \binom{L-1+k}{k} \frac{\left(1+\dfrac{2E_b}{LN_0}\right)^{k}}{\left(2+\dfrac{2E_b}{LN_0}\right)^{L+k}} . \tag{27}$$

B. FIRST ORDER SELECTION COMBINING (SC-1)

In first order selection combining (SC-1) the signal with the largest amplitude (hence the largest signal-to-noise ratio) is selected as shown Fig. 2.

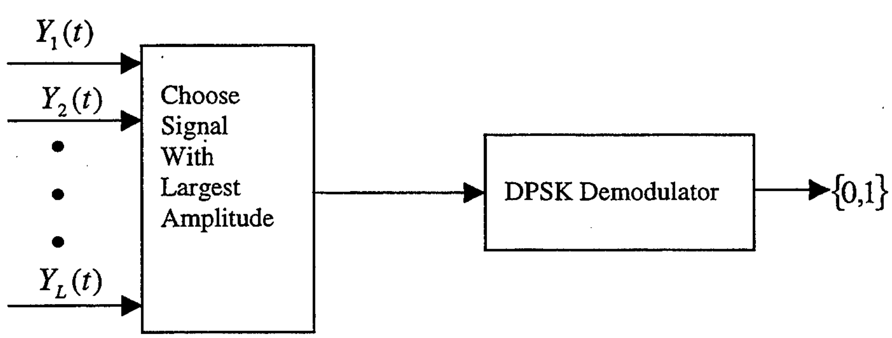

Figure 2. Selection Combining with L-th order Diversity

The conditional bit error probability for SC-1 is simply the bit error probability for DPSK conditional on the signal-to-noise ratio γ [4]

$$P_b(\gamma)=\frac{1}{2}\,e^{-\gamma}\ . \tag{28}$$

The pdf of γ is given by [3]

$$f(\gamma)=L\alpha\,e^{-\alpha\gamma}\left(1-e^{-\alpha\gamma}\right)^{L-1}, \tag{29}$$

where

$$\alpha=\frac{N_0}{\overline{E}}. \tag{30}$$

Here $\overline{E}$ is the average energy per diversity bit, and $\overline{E}/N_0$ is the average diversity bit energy-to-noise density ratio.

The bit error probability for SC-1 can be evaluated as follows [3]

$$P_b=\int_0^{\infty}P_b(\gamma)f(\gamma)d\gamma. \tag{31}$$

Inserting (28) and (29) into (31) and performing the integration, we obtain

$$P_b=\frac{L}{2}\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\,\frac{\alpha}{1+\alpha+k\alpha}. \tag{32}$$

We note that the average bit energy $\overline{E_b}$ is related to the average diversity bit energy $\overline{E}$ by $\overline{E_b}=L\overline{E}$. Thus using the identity $\alpha=\frac{LN_0}{\overline{E}_b}$, we obtain a result in terms of $\frac{\overline{E}_b}{N_0}$ as

$$P_b=\frac{L}{2}\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\,\frac{\left(\frac{LN_0}{\overline{E}_b}\right)}{1+\left(\frac{LN_0}{\overline{E}_b}\right)+k\left(\frac{LN_0}{\overline{E}_b}\right)}. \tag{33}$$

C. SECOND ORDER SELECTION COMBINING (SC-2)

In second order selection combining (SC-2), two signals with the largest amplitudes or signal-to-noise ratios at the input of the DPSK demodulator are combined. The bit error probability conditional on the signal-to-noise ratio γ is given by [3]

$$P_b(\gamma)=\frac{1}{8}e^{-\gamma}\left[4+\gamma\right]. \tag{34}$$

The pdf of γ is also given in [3]

$$f(\gamma)=L(L-1)\alpha\, e^{-\alpha\gamma}\left[\frac{\alpha\gamma}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left(1-e^{-\frac{\alpha k\gamma}{2}}\right)\right], \tag{35}$$

where

$$\alpha=\frac{N_0}{\overline{E}}, \tag{36}$$

as defined in (30). Substituting (34) and (35) into (31) yields

$$P_b=\int_0^\infty \frac{1}{8}e^{-\gamma}\left[4+\gamma\right]L(L-1)\alpha\; e^{-\alpha\gamma}\left[\frac{\alpha\gamma}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left(1-e^{-\frac{\alpha\gamma k}{2}}\right)\right]d\gamma\,. \tag{37}$$

By performing this integration, the bit error probability is found to be [3]

$$\begin{aligned}P_b=&\left(\frac{\alpha}{1+\alpha}\right)^2\frac{L(L-1)}{8}\\&\times\left\{2+\frac{1}{\alpha+1}+\sum_{k=1}^{L-2}\binom{L-2}{k}(-1)^k\left[\frac{(1+\alpha)\left[4(2+2\alpha+\alpha k)+4\right]+\alpha k}{(2+2\alpha+\alpha k)^2}\right]\right\}.\end{aligned} \tag{38}$$

The bit error probability is obtained in terms of $\frac{\overline{E}_b}{N_0}$ as

$$P_b = \left(\frac{\frac{LN_0}{\overline{E}_b}}{1+\frac{LN_0}{\overline{E}_b}} \right)^2 \frac{L(L-1)}{8} \left\{ 2 + \frac{1}{\frac{LN_0}{\overline{E}_b}+1} \right. \tag{39}$$

$$\left. + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k \left[\frac{\left(1+\frac{LN_0}{\overline{E}_b}\right)\left[4\left(2+2\frac{LN_0}{\overline{E}_b}+\frac{LN_0}{\overline{E}_b}k\right)+4\right]+\frac{LN_0}{\overline{E}_b}k}{\left(2+2\frac{LN_0}{\overline{E}_b}+\frac{LN_0}{\overline{E}_b}k\right)^2} \right] \right\}.$$

D. THIRD ORDER SELECTION COMBINING (SC-3)

For the third order selection combining (SC-3), three signals having the three largest amplitudes or signal-to-noise ratios at the input of the DPSK receiver are combined and the conditional bit error probability for DPSK is given by [3]

$$P_b(\gamma) = \frac{1}{32} e^{-\gamma} \left[16 + 6\gamma + \frac{1}{2}\gamma^2 \right]. \tag{40}$$

The pdf of γ for SC-3 is given by [3]

$$f(\gamma) = \frac{L(L-1)(L-2)}{2} \alpha e^{-\alpha \cdot \gamma} \tag{41}$$

$$\times \left\{ \frac{\alpha^2\gamma^2}{6} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[\alpha\gamma k - 3\left(1 - \exp\left(-\frac{k\alpha\gamma}{3}\right)\right)\right]\right\} .$$

Substituting (40) and (41) in (31) yields

$$P_b = \int_0^\infty \frac{1}{32} e^{-\gamma}\left[16+6\gamma+\frac{1}{2}\gamma^2\right]\frac{L(L-1)(L-2)}{2}\alpha\, e^{-\alpha\gamma} \times\left\{\frac{\alpha^2\gamma^2}{6}+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[k\alpha\gamma-3\left(1-e^{-\frac{\alpha\gamma k}{3}}\right)\right]\right\} d\gamma. \tag{42}$$

The bit error probability becomes [3]

$$\begin{aligned} P_b = \frac{L(L-1)(L-2)}{64}\Bigg\{ & \frac{16}{3\left(\frac{\overline{E}_b}{LN_0}+1\right)^3}+\frac{6\frac{\overline{E}_b}{LN_0}}{\left(\frac{\overline{E}_b}{LN_0}+1\right)^4}+\frac{2\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^5} \\ & +\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\Bigg[\frac{16k}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^2}-\frac{48}{\left(1+\frac{\overline{E}_b}{LN_0}\right)}+\frac{144}{\left(3+k+3\frac{\overline{E}_b}{LN_0}\right)} \\ & +\frac{12k\left(\frac{\overline{E}_b}{LN_0}\right)}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^3}-\frac{18\frac{\overline{E}_b}{LN_0}}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^2}+\frac{162\frac{\overline{E}_b}{LN_0}}{\left(3+k+3\frac{\overline{E}_b}{LN_0}\right)^2}+\frac{3k\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^4} \\ & -\frac{3\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^3}+\frac{81\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(3+k+3\frac{\overline{E}_b}{LN_0}\right)^3}\Bigg]\Bigg\}. \end{aligned} \tag{43}$$

In the next chapter we derive the bit error probability when first order post-detection selection combining is employed.

III. FIRST ORDER POST-DETECTION SELECTION COMBINING (PDSC-1) ANALYSIS

In the first-order Post-Detection Selection Combining (PDSC-1) technique [14], the maximum of L diversity samples at the output of the L noncoherent DPSK demodulators are considered. The diversity receiver is shown in Fig.3.

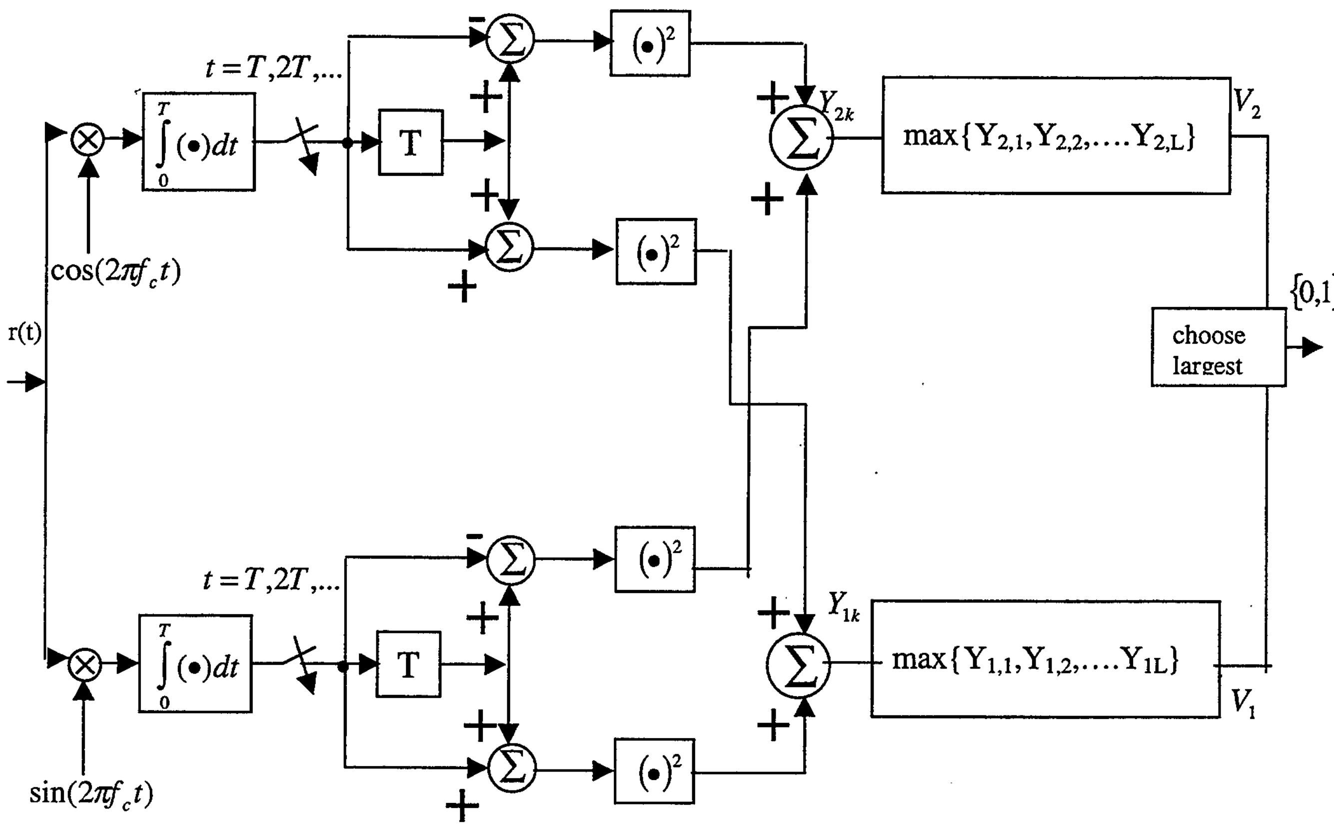

Figure 3. Block diagram of a noncoherent DPSK receiver with first order Post - Detection Selection Combining

A. PROBABILITY DENSITY FUNCTIONS OF DECISION VARIABLES

As shown in Fig. 3 V_1 and V_2 denote the decision variables for the signal branch and non-signal branch, respectively. We have

$$V_1 = \max\{Y_{1,1}, Y_{1,2} \ldots Y_{1,L}\}, \tag{44}$$

$$V_2 = \max\{Y_{2,1}, Y_{2,2} \ldots Y_{2,L}\}, \tag{45}$$

where Y_{1k} and Y_{2k}, $k = 1,2,\ldots L$ are the amplitudes at the output of the DPSK demodulator of the k-th diversity channel. We assume that Y_{1k} and Y_{2k} are independent and identically distributed random variables for all k.

For the signal branch, the probability density function of the k-th signal amplitude Y_{1k} is a central chi-square density function given in (20)

$$f_{Y_{1k}}(y_{1k}) = \frac{1}{2\sigma_1^2} \exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right), \qquad y_{1k} \geq 0, \tag{46}$$

where σ_1^2 is given in (21) and (22)

$$\sigma_1^2 = 4\sigma^2 + N_0 = 2\overline{E} + N_0 \ . \tag{47}$$

The probabilty density function of the decision variable V_1 in (44) is given by (Appendix A)

$$f_{V_1}(v_1) = L\ f_{Y_{1k}}(v_1)\left[F_{Y_{1k}}(v_1)\right]^{L-1}, \tag{48}$$

where

$$F_{Y_{1k}}(v_1) = \int_0^{v_1} \frac{1}{2\sigma_1^2} \exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right) dy_{1k} = 1 - \exp\left(-\frac{v_1}{2\sigma_1^2}\right). \tag{49}$$

Substituting (46) and (49) into (48) yields